Quick Review

More on independence:

- A, B independent $\iff \mathbb{P}[A \cap B] = \mathbb{P}[A]\mathbb{P}[B]$
  $\iff \mathbb{P}[A \mid B] = \mathbb{P}[A]$

- $A_1, \dots, A_K$ are <u>mutually independent</u> if for all subsets $I \subseteq \{1, \dots, K\}$,

$$\mathbb{P}\left[\bigcap_{i \in I} A_i\right] = \prod_{i \in I} \mathbb{P}[A_i]$$

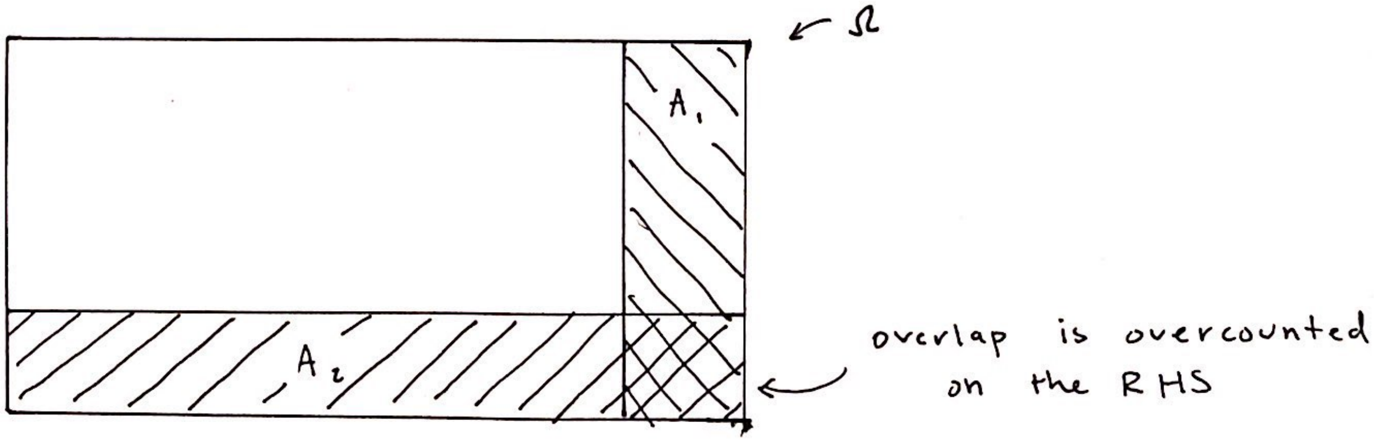
- Equivalently, mutually independent if for all $B_i \in \{A_i, \overline{A_i}\}$ $i = 1, \dots, K$,

$$\mathbb{P}[B_1 \cap \cdots \cap B_K] = \prod_{i=1}^{K} \mathbb{P}[B_i]$$

- MI is <u>much stronger</u> than pairwise independence
  - ↳ PWI: $O(n^2)$ constraints, vs.
  - ↳ MI: $O(2^n)$ constraints.

Union Bound:

- $\mathbb{P}\left[\bigcup_i A_i\right] \leq \sum_i \mathbb{P}[A_i]$



Useful Things

- $\mathbb{P}[A \cap B] = \mathbb{P}[A|B]\,\mathbb{P}[B] = \mathbb{P}[B|A]\,\mathbb{P}[A]$
- (Bayes') $\mathbb{P}[A|B] = \dfrac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]} = \dfrac{\mathbb{P}[A]\,\mathbb{P}[B|A]}{\mathbb{P}[B]}$
- (Total Prob.) $\mathbb{P}[A] = \mathbb{P}[A \cap B] + \mathbb{P}[A \cap \bar{B}]$

- Start w/ what you <u>want</u>, transform into what you <u>have</u>.
- Turn expressions into words and vice versa
- Draw a picture (esp. useful in Bayes' problems)
- Some tips for this:
  - ↳ If you only have terms involving A, B (e.g. $\mathbb{P}[A \cap B]$) but want to isolate, think total probability.
  - ↳ If you want to swap order of conditionals (e.g. $\mathbb{P}[A|B] \longrightarrow \mathbb{P}[B|A]$), think Bayes'
  - ↳ See an inequality $\Rightarrow$ Union Bound.