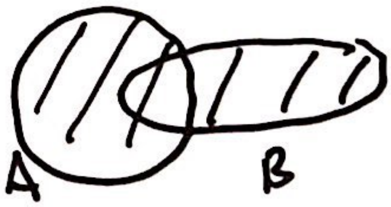
Quick Review

$A \cup B$, <u>union</u>

consists of all elements in A <u>or</u> in B

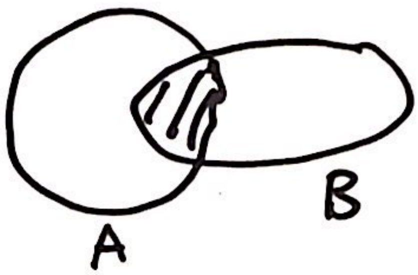
$\cup \approx \vee$



$A \cap B$, <u>intersection</u>

consists of all elements in A <u>and</u> B

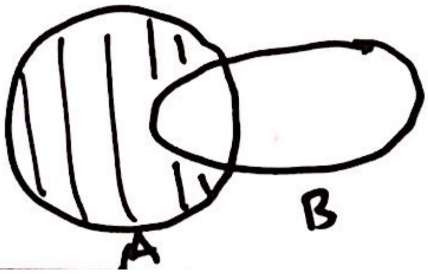
$\cap \approx \wedge$



$A \setminus B$, difference

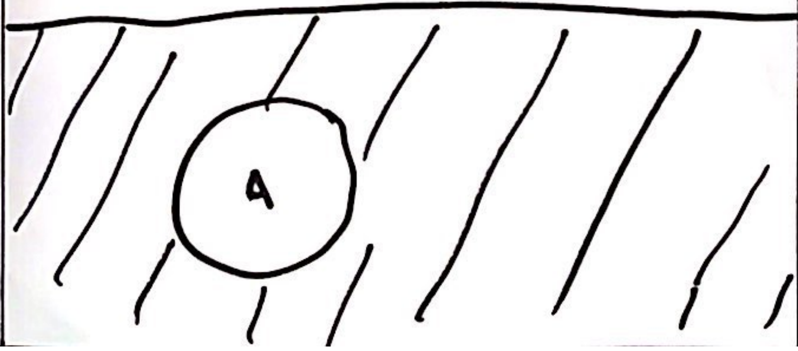
$A - B$

consists of all elements in A <u>but not in</u> B



$\bar{A}$, complement

First specify the universe U, the set that consists of everyth- in U not in A.



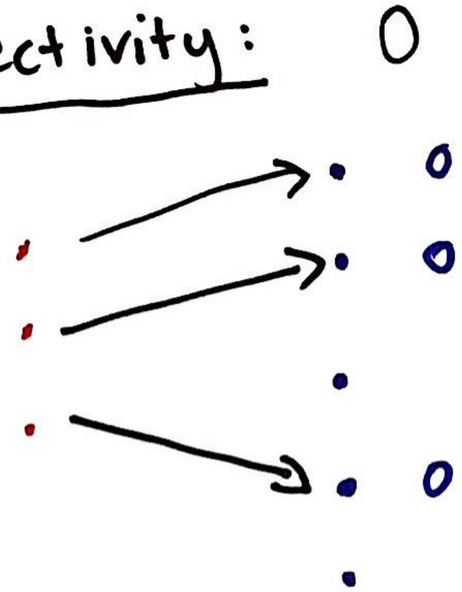
$f : A \to B$

domain

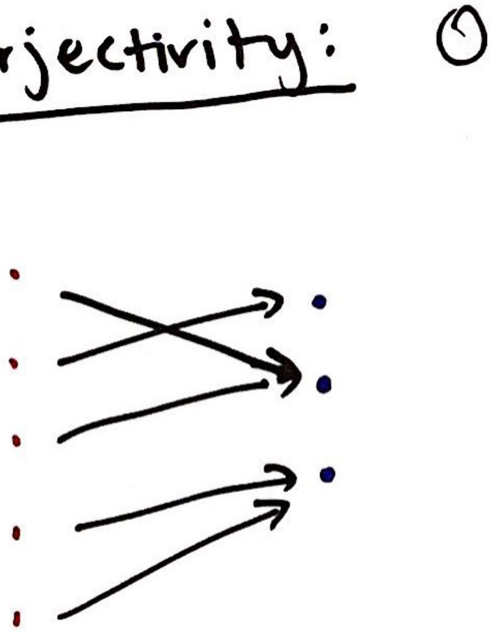
codomain

$\neq$ range

injectivity:



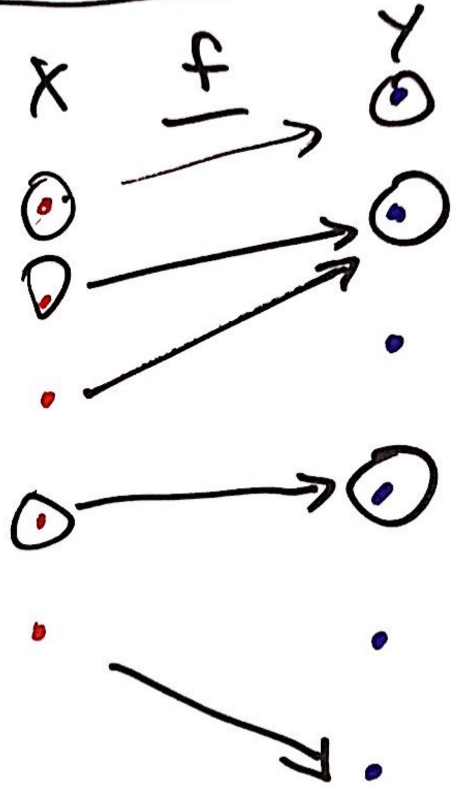
surjectivity:



bijectivity

- $f : A \to B$

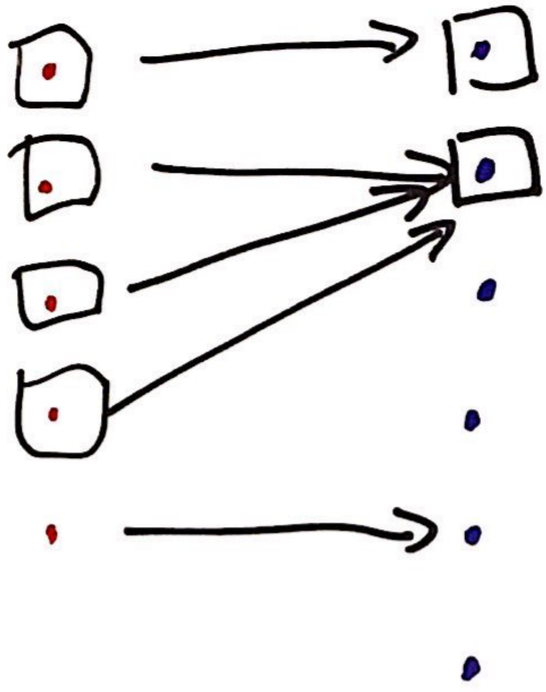
$|A| = |B|$



the *image* of a set $A \subseteq X$ is the set

$f(A) = \{ f(a) \mid a \in A \}$

the <u>pre-image</u> of a set $B \subseteq Y$ is

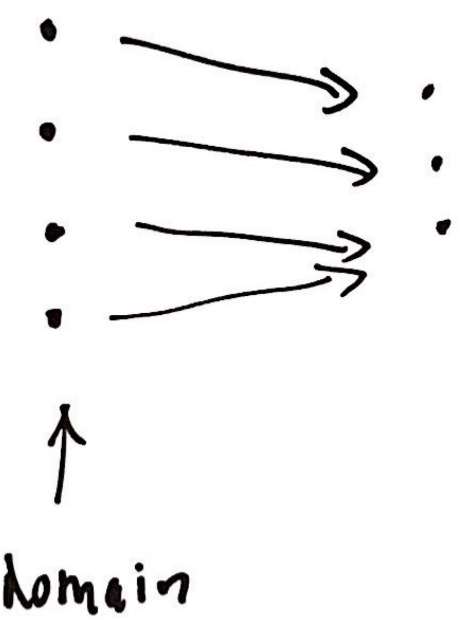


$$f^{-1}(B) = \{ a \mid a \in X,\ f(a) \in B \}$$

$f(x) = \frac{1}{x}$ a function?

$f : \mathbb{R} \to \mathbb{R}$ then <u>no</u>.

$f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ then <u>yes</u>.



$f(2)$ can't be two things.

X domain

Y codomain

$\forall x \in X$, $f(x)$ exists and is unique.

Discussion 1D Problem 1

(a) If $A = \{1, 2, 3, 4\}$, what is $\mathcal{P}(A)$?

The set of all subsets of $\{1, 2, 3, 4\}$, which is tedious to write out.

(b) If B is a set, describe $\mathcal{P}(B)$ using set comprehension.

$\{b \mid b \subseteq B\}$.

(c) What is $\mathbb{R} \cap \mathcal{P}(A)$?

ϕ, as the elements of $\mathbb{R}$ are numbers and the elements of $\mathcal{P}(A)$ are sets

(d) What is $\mathbb{R} \cap \mathbb{Z}$?

$\mathbb{Z}$, since the integers are a subset of the reals.

(e) What is in $\mathbb{N} \cup \mathbb{Q}$?

$\mathbb{Q}$, since the naturals are a subset of the rationals.

(f) What kind of numbers are in $\mathbb{R} \setminus \mathbb{Q}$?

Irrational numbers.

(g) If $S \subseteq T$, what is $S \setminus T$.

ϕ, as T contains all of S.

Discussion 1D Problem 2

Let X, Y be sets, and $f : X \longrightarrow Y$ be a function.

- If $A \subseteq X$, define the <u>image</u> of A under f as the set

$$f(A) := \{ f(x) \mid x \in A \}.$$

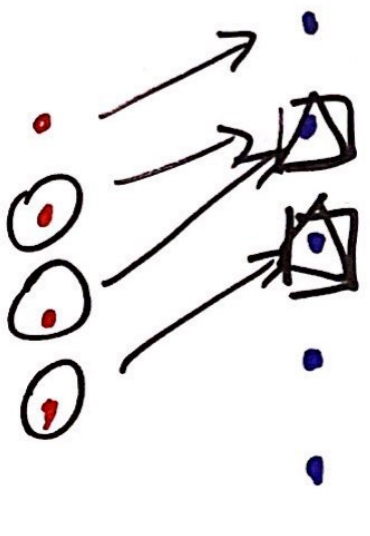
- If $B \subseteq Y$, define the <u>pre-image</u> of B under f as the set

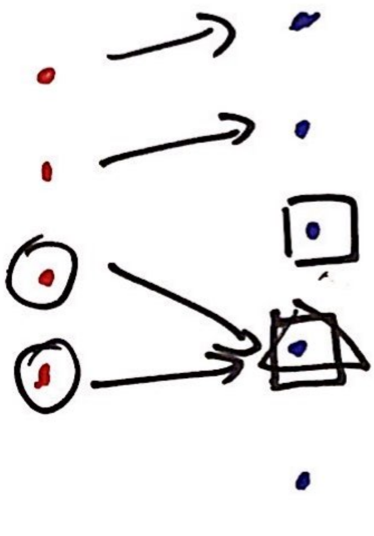
$$f^{-1}(B) := \{ x \mid x \in X \wedge f(x) \in B \}.$$

(a) If $B \subseteq Y$, prove that $f(f^{-1}(B)) \subseteq B$.

Idea: prove that ~~$\forall b \in B$,~~

$\forall b \in f(f^{-1}(B))$, $b \in B$





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Idea: Prove $\forall b \in f(f^{-1}(B))$, $b \in B$

Let $b \in f(f^{-1}(B))$ be arbitrary.
Then ~~f(b)~~ $\exists\, a \in f^{-1}(B)$ such that $f(a) = b$. Since $a \in f^{-1}(B)$, $f(a) \in B$. $b = f(a) \in B$.

(b/c) Let $A \subseteq X$. Prove that $A \subseteq f^{-1}(f(A))$ and provide an example where $A \neq f^{-1}(f(A))$

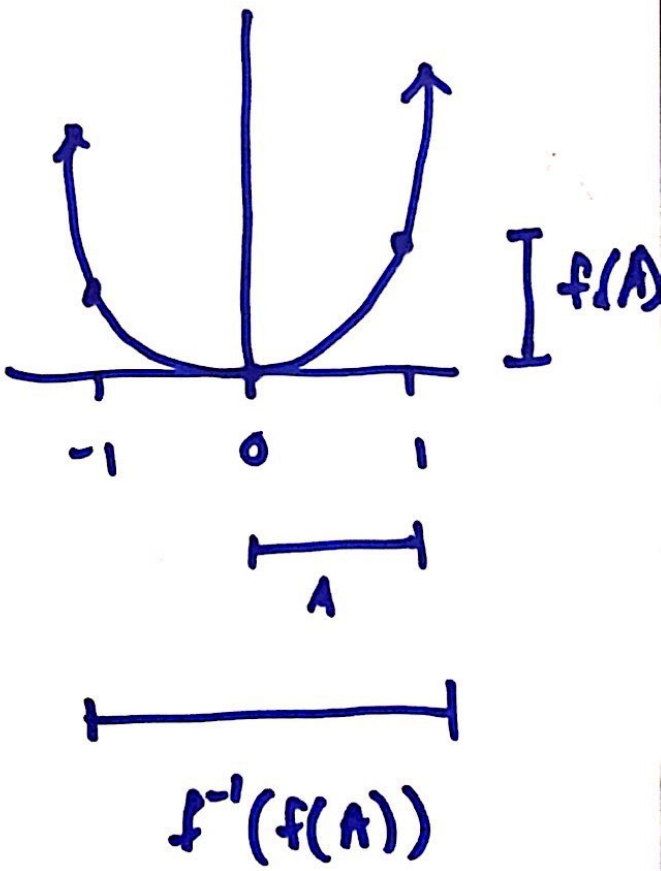
Idea: $\forall a \in A$, prove that $a \in f^{-1}(f(A))$

Since $a \in A$, it follows that $f(a) \in f(A)$. By the definition of the preimage, $a \in f^{-1}(f(A))$ because $f(a) \in f(A)$.

$f(x) = x^2$, $f: \mathbb{R} \to \mathbb{R}$.

$A = [0,1]$.

$f(A) = [0,1]$



Discussion 1D Problem 3

Consider the function

$$f(x) = \begin{cases} x & \text{if } x \geq 1 \\ x^2 & \text{if } -1 \leq x < 1 \\ 2x+3 & \text{otherwise.} \end{cases}$$

(a) If the domain / codomain of f are $\mathbb{N}$, is f

- Injective?

  Yes, as on $\mathbb{N}$, $f(x) = x$, so if $f(a) = f(b)$, then $a = b$.

- Surjective?

  Yes. If $n \in \mathbb{N}$ is arbitrary, then $f(n) = n$, so f can reach any element in $\mathbb{N}$.

- Bijective?

  Yes, as it is both ~~bijective~~ injective and surjective.

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(b) If the domain / codomain of f are $\mathbb{Z}$, is f

- injective?

  No, as $f(1) = f(-1) = 1$.

- surjective?

  No, as there is no $x \in \mathbb{Z}$ such that $f(x) = -2$.

- Bijective?

  No, as it is neither injective nor surjective.

If the domain / codomain of f are $\mathbb{R}$, is f

- injective?

  No, as again $f(1) = f(-1) = 1$.

- surjective?

  Yes. Let $r \in \mathbb{R}$ be arbitrary. Then
  - if $r \geq 1$, $f(r) = r$,
  - if $0 \leq r < 1$, $f(\sqrt{r}) = r$, and
  - if $r < 0$, $f\left(\frac{r-3}{2}\right) = r$,

  so $\forall r$ we can find x such that $f(x) = r$.

- bijective?

  No, as it is not injective.