

## Important Facts

- If  $G = (V, E)$  is a graph, the following statements are equivalent. (Let  $n = |V|$ )
  - (1)  $G$  is a tree.
  - (2)  $G$  is connected and acyclic.
  - (3)  $G$  is connected and has  $n-1$  edges.
- If  $G = (V, E)$  is a graph, the following are also equivalent.
  - (1)  $G$  is complete.
  - (2)  $G$  has  $n(n-1)/2$  edges.
  - (3) ~~✗~~ Every vertex in  $G$  has degree  $n-1$ .
- Hypercubes in  $n$  dimensions . . .
  - Have  $2^n$  vertices
  - Have  $n \cdot 2^{n-1}$  edges
  - Are bipartite

True or False :

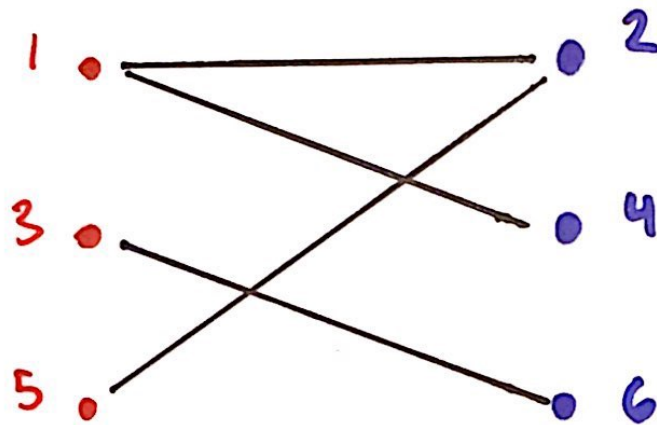
2B # 1

- (a) Any pair of vertices in a tree are connected by exactly one path.
- (b) Adding an edge between two vertices of a tree creates a cycle.
- (c) Adding an edge in a connected graph creates exactly one new cycle.

# Bipartite Graph

| 2B # 2

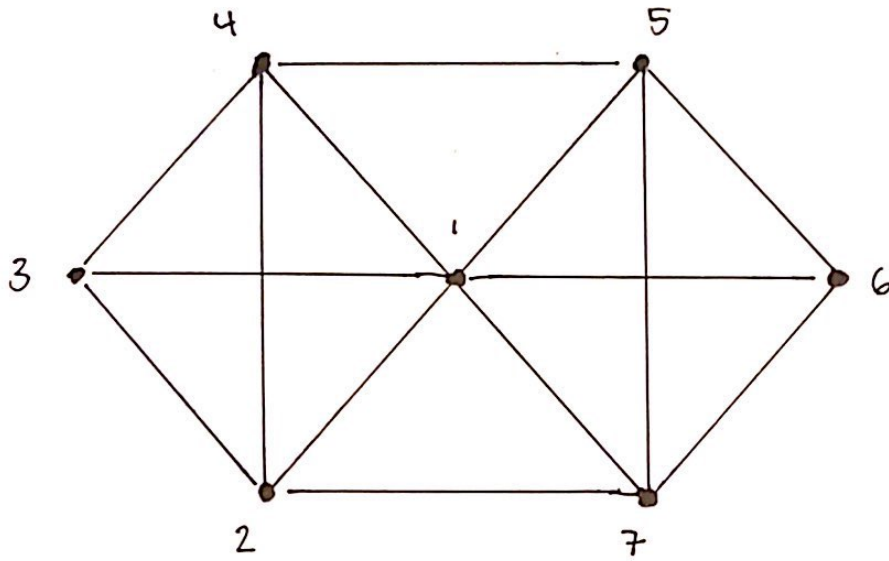
A graph is bipartite if we can split its vertices into two nonempty sets such that every edge in the graph connects vertices in different sets.



Prove that a bipartite graph contains no tours of odd length.

# Eulerian Tour / Walk

2B # 3



(a) Does the graph have an Euler Tour?

(b) Does it have an Euler walk?

## Odd Degree Vertices

2B # 4

Let  $G = (V, E)$  be an undirected graph.  
Prove that  $G$  has an even number of  
vertices with odd degree.